

the other hand, a great variety of pretty inequalities are offered: the Bernstein–Szegő inequality, Videnskii’s inequalities, inequalities for entire functions of exponential type, Markov’s inequality for higher derivatives, and weighted versions of the Bernstein and Markov inequalities. These inequalities are extended to Müntz spaces in Chapter 6, where Newman’s inequality is the basic result, and to rational function spaces in Chapter 7.

At the end there are five appendices dealing with some additional material such as algorithms and computational complexity for polynomials and rational functions (Appendix A1); orthogonality and irrationality, with a proof of the irrationality of $\zeta(3)$, π^2 , and $\log 2$ (Appendix A2); an interpolation theorem for linear functionals (Appendix A3); recent material on inequalities for generalized polynomials in L^p (Appendix A4); and inequalities for polynomials with constraints (Appendix A5).

This is a wonderful book which is strongly recommended for use in a class with students who are willing to work on the proofs, rather than to digest fully prepared and worked out proofs and examples. I have already used Appendix A2 successfully in a few research seminars and believe that the material in the book and the approach taken by the authors will prove to be a success.

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K. B. Datta and B. M. Mohan, *Orthogonal Functions in Systems and Control*, Advanced Series in Electrical and Computer Engineering 9, World Scientific, Singapore, 1995, xi + 275 pp.

Although a part of this book deals with the sine and cosine functions and with piecewise linear functions such as Haar, Rademacher, Walsh, and block pulses, the main emphasis is on traditional orthogonal polynomials from Legendre to Gegenbauer.

The fundamental role played by such orthogonal function systems in approximation theory and numerical analysis needs no advertising. They have a history going back to the 18th and 19th centuries. Their use in signal processing, differential equations, systems, and control is somewhat less widespread and much more recent. The early applications appear as late as the 1970s. The first chapter of this book is mainly an extensively annotated bibliography about the role these orthogonal systems played in those applications from about 1970 to 1990.

The second chapter gives a collection of all the basic properties of the orthogonal functions. The next chapter moves towards signal processing by discussing shifted versions of the polynomials, their approximating properties in the presence of noise, treatment of two-dimensional signals, and the effect of differentiation and integration. The latter leads to quadrature formulas by specifying how the integral of the basis functions is expressed in terms of the same set of basis functions.

Thus the actual part dealing with systems and control starts about halfway through the book with Chapter 4. In time-delay systems the state vector x and the control vector u are coupled by $\dot{x}(t) = Ax(t) + Bu(t) + Fx(t - \tau) + Gu(t - \tau)$, where τ is the delay, and A , B , G and F are matrices. For the solution of such systems, a framework of integration and delay integration is developed along the lines of the previous chapter.

The next three chapters deal with identification problems, i.e., to estimate parameters and possibly initial conditions and boundary conditions for a system. One has to solve a differential equation and the basic mathematical problem is to approximate repeated integration in “one shot” by a linear combination of the functions in the orthogonal system. The last chapter solves a control problem. That is, the optimal control vector u is written as $u(t) = K(t)x(t)$, where x is the state vector and $K(t)$ the gain matrix to be computed.

The book deals with an area that is not the subject of widespread research interest. At no point did the book surprise me, nor did it offer new perspectives. Being conceived as lecture notes (with exercises) it was probably not intended to have these effects. It could be of interest to students and researchers in approximation theory, systems theory, or numerical analysis (all the methods are illustrated with numerical examples). Certainly not all the possibilities have been explored and the methods proposed may be considered as samples of the potential applicability of orthogonal function systems in this area. It is unfortunate, however, that more exciting modern orthogonal systems such as wavelets are not even mentioned.

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T. Ransford, *Potential Theory in the Complex Plane*, London Mathematical Society Student Texts **28**, Cambridge Univ. Press, Cambridge, UK, 1995, x + 232 pp.

This is a marvelously written text containing the fundamentals of logarithmic potentials. Although there are many monographs on potential theory, very often the two-dimensional theory is treated (if treated at all) somewhat left-handedly as the limit case of more general kernels, while the theorems are mentioned in passing as exceptions to the general, in most cases rather abstract, theory. This should not be so, for the two-dimensional theory was the very model for the more abstract developments and it has an extremely strong relationship with complex analysis, as is beautifully illustrated in this book. This is the first of its kind in the sense that it concentrates on the two-dimensional theory, allowing students to discover the beauty and utility of the theory without too much background or too many abstractions. From the preface: "Indeed it was consciously written as the book that I should have liked to read all those years ago."

The book starts out with the basics of harmonic functions: mean value property, maximum principle, Poisson integral, and Harnack's inequality. This chapter culminates in the recent proof of Picard's theorem by J. Lewis. The next chapter is on subharmonic functions and the maximum principle, the latter of which is used to prove several forms of the Phragmén-Lindelöf principle.

Chapter 3 deals with logarithmic potentials, extremal measures, and polar sets. Here the natural applications are in connection with removable singularities. In fact, this is the first example that shows the relevance of potential theory to function theory. Polar sets are the exceptional sets of potential theory much like sets of zero measure are the exceptional sets in measure theory. Polar sets are where logarithmic potentials can take the value infinity, and yet they pop up also as sets of removable singularities: if a function on a region G is locally bounded and holomorphic outside a polar set, then it can be uniquely extended to a holomorphic function on G (this is only the simplest form of the principle).

Chapter 4 deals with the Dirichlet problem. The geometric structure of the boundary of the domain plays an important role, so the so-called regularity property of boundary points forms a central part in the theory. In this chapter the analytic tools, such as harmonic measures, Green functions, and the Poisson-Jensen formula, are already in full swing. The main applications in this chapter are Lindelöf's theorem on asymptotic values of holomorphic functions (if f is bounded and holomorphic on the upper half-plane and it has a limit value Z along a curve tending to infinity, then $f(z) \rightarrow Z$ uniformly as $z \rightarrow \infty$ in such a way that $\arg(z)$ lies between two constants $\varepsilon > 0$ and $\pi - \varepsilon$), the Riemann mapping theorem, and continuity properties of conformal mappings at boundary points.

Chapter 5 is devoted to properties of logarithmic capacity, transfinite diameter, and Wiener's criteria for continuity of logarithmic potentials. Finally, Chapter 6 contains applications in five different directions. The first one is the Riesz-Thorin interpolation theorem on the